

# A statistical approach to distinguish telomere elongation from error in longitudinal datasets

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**Abstract** Telomere length and the rate of telomere attrition vary between individuals and have been interpreted as the rate at which individuals have aged. The biology of telomeres dictates shortening with age, although telomere elongation with age has repeatedly been observed within a minority of individuals in several populations. These findings have been attributed to error, rather than actual telomere elongation, restricting our understanding of its possible biological significance. Here we present a method to distinguish between error and telomere elongation in longitudinal datasets, which is easy to apply and has few assumptions. Using simulations, we show that the method has considerable statistical power (>80 %) to detect even a small proportion (6.7 %) of TL increases in the population, within a relatively small sample ( $N = 200$ ), while

maintaining the standard level of Type I error rate ( $\alpha \leq 0.05$ ).

**Keywords** Telomere length · Statistics · Telomere shortening · Within individual · Aging · Human

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Telomeres are DNA sequence repeats at the end of chromosomes. These repeats shorten at each cell replication or by damage, and critical telomere lengths lead to cellular senescence, apoptosis and/or genome instability (Riethman 2008). These properties of telomeres suggest direct involvement in aging mechanisms, but telomere length (TL) may also be an indicator of the progression of aging within individuals and/or differences in aging between individuals (Mather et al. 2011; Riethman 2008). Indeed, short TL is associated with higher mortality risk in humans (Boonekamp et al. 2013) and other free-living animals (e.g. Barrett et al. 2013; Bize et al. 2009; Heidinger et al. 2012; Salomons et al. 2009). Yet, comparative analyses do not support that shorter telomeres dictate shorter lifespans between species (Gorbunova and Seluanov 2009). The rate at which telomeres shorten is also variable between individuals (e.g. Aviv et al. 2009; Nordfjäll et al. 2009) and higher rates of telomere attrition are associated with increased risk of mortality (Epel et al. 2009).

The biological properties of telomeres dictate shortening rather than lengthening in tissues in which

telomeres are not actively elongated (Gorbunova and Seluanov 2009). Yet in the majority of studies TL increases are apparent within a small group of individuals. These elongations are often attributed to error (e.g. Aviv et al. 2009; Beaulieu et al. 2011; Bize et al. 2009; Chen et al. 2011; Ehrlenbach et al. 2009; Epel et al. 2009; Foote et al. 2011; Nordfjäll et al. 2009; Salomons et al. 2009; Shalev et al. 2012; Steenstrup et al. 2013) which is composed of both measurement error of TL and other unknown causes of within-individual variability (e.g. variation in TL of the tissue sampled). An alternative explanation is that telomeres do elongate in some individuals. To our knowledge, no statistical approach exists to distinguish telomere elongation from error within longitudinal studies. Here we present a method, which is easy to apply and has few assumptions. Using simulations, we show that this method has considerable statistical power (>80 %), while it retains the standard level of Type I error rate ( $\alpha \leq 0.05$ ).

Our method first requires estimating variance due to measurement errors (error variance) in two distinct ways related to two different assumptions: (1) TL increases and/or decreases and (2) telomeres do not elongate. Under the first assumption, error variance can be estimated in two steps. First, we estimate the residual variance for each individual using an ordinary (least square) linear regression:

$$y_i = \beta_0 + \beta_1 t_i + \varepsilon_i, \quad (1)$$

$$\varepsilon_i \sim N(0, \sigma_\varepsilon^2), \quad (2)$$

where  $t_i$  is the  $i$ th time point at which TL,  $y_i$ , is measured ( $i = 1, 2, \dots, n$ ;  $n$  is the number of TL measurements and  $n > 2$ ),  $\beta_0$  is the intercept (TL at  $t = 0$ ),  $\beta_1$  is the slope (regression coefficient for  $t$ ), and  $\varepsilon_i$  is the  $i$ th residual value. Residuals are normally distributed ( $N$ ) with a variance of  $\sigma_\varepsilon^2$ . If  $\sigma_{\varepsilon_j}^2$  represents the  $j$ th individual's residual variance ( $j = 1, 2, \dots, N$ ;  $N$  is the number of individuals in a study), then, an overall error variance estimate of TL ( $\bar{\sigma}_\varepsilon^2$ ) can be obtained by taking an average of  $\sigma_{\varepsilon_j}^2$ :

$$\bar{\sigma}_\varepsilon^2 = \frac{1}{N} \sum_{j=1}^N \sigma_{\varepsilon_j}^2, \quad (3)$$

Perhaps, more practically, Eq. 3 can be re-written using the residual sum of squares:

$$\bar{\sigma}_\varepsilon^2 = \frac{1}{N} \sum_{j=1}^N \frac{1}{n_j - 2} \sum_{i=1}^{n_j} \varepsilon_{ij}^2, \quad (4)$$

where  $n_j$  is the number of TL measurements  $n$  for  $j$ th individual and  $\varepsilon_{ij}^2$  is the squared residual value for the  $i$ th time point for the  $j$ th individual (cf. Crawley 2005).

Under the second assumption (i.e. no telomere elongation), the measurement error variance ( $\sigma_\varepsilon^2$ ) can be obtained by:

$$\sigma_\varepsilon^2 = \frac{1}{2(m-1)} \sum_{k=1}^m D_k^2, \quad (5)$$

where  $D_k^2$  is the difference in TL between the initial and last measurements in the  $k$ th individuals that showed an increase in TL ( $k = 1, 2, \dots, m$ ;  $m$  is the number of individuals whose TL elongated). When observed TL increases are not due to error, but consistent telomere elongation is present in the population, the largest increases of TL are between the first and the last measurement in time. Therefore to increase sensitivity of detecting telomere elongation we define telomere increases as the TL at the last measurement minus the TL at the first measurement per individual as in Eq. 5. Note that the same equations can be used to ask the question whether telomere increases occur at any point in time in the population. A mathematical derivation of Eq. 5 is given in the [Appendix](#).

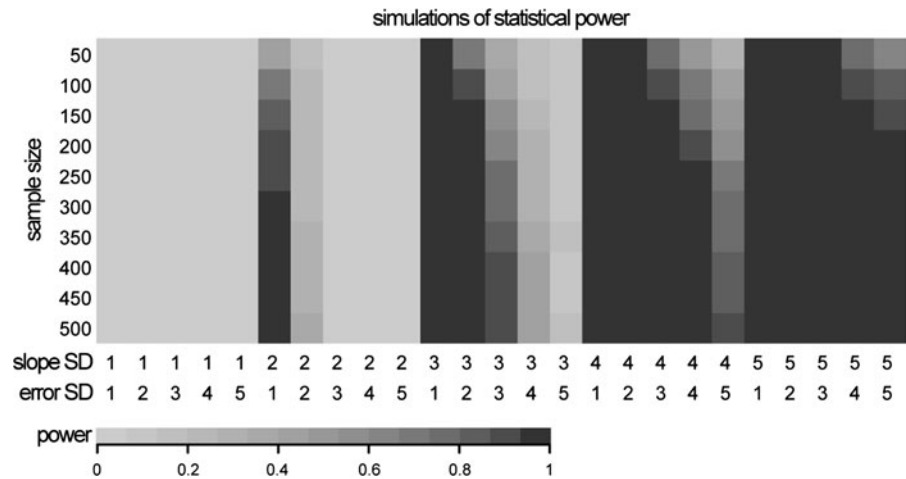
When the estimated error variance  $\sigma_\varepsilon^2$  (Eq. 5) is larger than the error variance  $\bar{\sigma}_\varepsilon^2$ , when TL is allowed to increase or decrease (Eq. 4), the hypothesis that telomeres show no elongation in the sample can be rejected. Statistically, such a comparison can be achieved using a variance ratio test between  $\sigma_\varepsilon^2$  and  $\bar{\sigma}_\varepsilon^2$ . The ratio of these two estimated error variances should follow an  $F$  distribution, which can be written as:

$$\frac{\sigma_\varepsilon^2}{\bar{\sigma}_\varepsilon^2} \sim F(m-1, N-1) \quad (6)$$

where the  $F$  distribution is defined by two degrees of freedom (DF): (1) the numerator DF is the number of observed TL increases minus 1 ( $m-1$ ), and (2) the denominator DF is the number of individuals in a study minus 1 ( $N-1$ ) (Crawley 2007).

When telomere elongation is statistically detected within a population, the identification of individuals

**Fig. 1** Result of the statistical power simulations. The statistical power (indicated by the grayscale, darker means higher power, the fraction of times the null hypothesis is rejected when it is actually false) is dependent on the sample size on the y-axis, and the error standard deviation (error SD) and slope standard deviation (slope SD), both depicted on the x-axis



within the population that are likely to show true telomere elongation (i.e. not the resultant of measurement errors) can be identified using the upper confidence limit (UCL) of  $\bar{\sigma}_e^2$  (Crawley 2007). The UCL of the 95 % confidence interval (note that the 95 % here is rather arbitrary and can be changed depending on the level of certainty required) can be written as:

$$97.5\% \text{ UCL} = \frac{(N-1)\bar{\sigma}_e^2}{\chi_{N-1(0.975)}^2} \quad (7)$$

where  $\chi_{N-1(0.975)}^2$  is the value at  $p = 0.975$  of the  $\chi^2$  distribution defined by  $DF = N - 1$ . This UCL of  $\bar{\sigma}_e^2$  can be used to determine the normal distribution of the UCL of the underlying measurement error distribution. Subsequently individual telomere increases (note that the increases should be divided by 2 as in Eq. 5, because the TL increases are a result from the addition of two equal error distributions) that are at the boundary of this normal distribution (with e.g. 95 % confidence) can be looked up with, for example, the function 'qnorm' (Wichura 1988) from R (R Development Core Team 2011). These specific individuals can be selected for follow-up studies, to examine biological and environmental correlates (see also the worked example provided with the manuscript).

To investigate the statistical power of the approach proposed here, simulations were conducted in R (code is available upon request). Individual based data of three time points per individual were generated. Individuals were set to lose an average TL of three units per time, which varied among individuals with a given standard deviation (labeled slope SD). At each

time point TL was subject to error (labeled error SD). Simulations were run for different combinations of sample size (range 50–500), error and slope SDs (both range 1–5) and each simulation was run 1,000 times. The resulting statistical power was calculated as the fraction of times the null hypothesis was rejected when it was actually false (Fig. 1), in other words, if the method detected telomere elongation when true telomere elongation was present in the simulated data. As expected, power increased with lower error, larger sample size and higher incidence of telomere elongation in the sample (i.e. higher slope SD). The average proportion of individuals showing a 'real' positive slope in these simulations was 0.13, 6.7, 16, 23 and 27 % for slope SD 1, 2, 3, 4, 5 respectively. Note that 0.13 % might not be a biologically relevant proportion of individuals that show true telomere elongation, yet in the continuum presented in Fig. 1, it does give an impression of sensitivity and reliance on outliers of the method presented, and for this reason we included it in our power simulations. The statistical approach presented here is thus able to detect telomere elongation of only a small proportion ( $\geq 6.7$  %) of a relatively small sample (under 500 individuals) with considerable power. In addition, the chance of rejecting the null hypothesis when it is actually true (Type I error) was simulated using a slope SD of 0 and without an average decrease in TL for a range of sample sizes (50–500) and error SD (of 3 and 4). Type I error rates were equal to the expected  $\alpha$ , 5 % (4.8 % of 10,000 simulations) and were independent of sample size and error SD. Note that if there is an average decrease of TL across the population, type I error rates will be

much lower given that the decline of TL over time reduces the amount of increases due to error.

The formal test to distinguish true telomere elongation from error, described here, forms an incentive to measure individuals at least three times longitudinally.

The detection of significant elongation of TL within a population will likely spur research into the mechanisms regulating telomere elongation and into specific properties or circumstances of the individuals that show true telomere elongation.

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## Appendix

The derivation of Equation 5

A two-level regression which model telomere length (TL) can be expressed as:

$$y_{ij} = \beta_0 + \gamma_j + (\beta_1 + \varphi_j)t_{ij} + \varepsilon_{ij}, \quad (\text{A1})$$

$$\begin{pmatrix} \gamma_j \\ \varphi_j \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\gamma^2 & \rho\sigma_\gamma\sigma_\varphi \\ \rho\sigma_\gamma\sigma_\varphi & \sigma_\varphi^2 \end{pmatrix}\right), \quad (\text{A2})$$

$$\varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2) \quad (\text{A3})$$

where  $t_{ij}$  is the  $i$ th time point at which TL,  $y_{ij}$  is measured for the  $j$ th individual ( $i = 1, 2, \dots, n$ ;  $n$  is the number of TL measurements and  $n > 2$ ;  $j = 1, 2, \dots, N$ ;  $N$  is the number of individuals in a study),  $\beta_0$  is the grand intercept (TL at  $t = 0$ ),  $\beta_1$  is the grand slope (regression coefficient for  $t$ ),  $\gamma_j$  is the deviation from  $\beta_0$  for the  $j$ th individual,  $\varphi_j$  is the deviation from  $\beta_1$  for the  $j$ th individual,  $\gamma_j$  and  $\varphi_j$  has a multivariate normal distribution with the variance–covariance structure specified in A2, and  $\varepsilon_{ij}$  is the  $i$ th residual value and residuals are normally distributed with a variance of  $\sigma_\varepsilon^2$ .

When we consider A1 at the time points 1 and  $n$  (i.e.  $i = 1$  and  $i = n$ ), TL can be written as:

$$y_{1j} = \beta_0 + \gamma_j + (\beta_1 + \varphi_j)t_{1j} + \varepsilon_{1j}, \quad (\text{A4})$$

$$y_{nj} = \beta_0 + \gamma_j + (\beta_1 + \varphi_j)t_{nj} + \varepsilon_{nj}. \quad (\text{A5})$$

When we have two measurements in time, 1 and  $m$  (the final time point) of telomeres the difference in telomere length is described by:

$$y_{nj} - y_{1j} = \beta_1(t_{nj} - t_{1j}) + \varphi_j(t_{nj} - t_{1j}) + \varepsilon_{nj} - \varepsilon_{1j}. \quad (\text{A6})$$

By setting  $d_j = y_{nj} - y_{1j}$ , the variance of  $d_j$  can be expressed as:

$$\text{Var}(d_j) = (t_{nj} - t_{1j})^2\sigma_\varphi^2 + 2\sigma_\varepsilon^2. \quad (\text{A7})$$

Note that the constant  $\beta_1(t_{nj} - t_{1j})$  disappears. Using the definition of variance and further rearranging:

$$\frac{1}{(N-1)} \sum_{j=1}^N (d_j - \bar{d})^2 = (t_{nj} - t_{1j})^2\sigma_\varphi^2 + 2\sigma_\varepsilon^2, \quad (\text{A8})$$

$$\frac{1}{2(N-1)} \sum_{j=1}^N d_j^2 = \sigma_\varepsilon^2 + \frac{(t_{nj} - t_{1j})^2\sigma_\varphi^2}{2} + \frac{\bar{d}^2}{2}, \quad (\text{A9})$$

where  $\bar{d}$  is the mean value of  $d_j$ . As  $\bar{d} = \beta_1(t_{nj} - t_{1j})$  and setting  $(t_{nj} - t_{1j}) = u$ ;

$$\frac{1}{2(N-1)} \sum_{j=1}^N d_j^2 = \sigma_\varepsilon^2 + \frac{u^2}{2}(\sigma_\varphi^2 + \beta_1^2). \quad (\text{A10})$$

When we assume that TL does not increase or decrease, i.e.  $(\sigma_\varphi^2 + \beta_1^2) = 0$ , A10 reduces to:

$$\sigma_\varepsilon^2 = \frac{1}{2(N-1)} \sum_{j=1}^N d_j^2. \quad (\text{A11})$$

If we estimate  $\sigma_\varepsilon^2$  in A11 only from individuals that show an increase of TL, or  $d_j > 0$  (set such  $d_j$  as  $D_j$ ), we have Eq. 5 from the main text;

$$\sigma_\varepsilon^2 = \frac{1}{2(m-1)} \sum_{k=1}^m D_k^2, \quad (\text{A12})$$

where  $D_k^2$  is the difference in TL between the initial and last measurements in the  $k$ th individuals that showed an increase in TL ( $k = 1, 2, \dots, m$ ;  $m$  is the number of individuals whose TL elongated). Note that we assume  $\sigma_\varepsilon^2$  is also normally distributed as with  $\sigma_\varepsilon^2$  (A3). Due the symmetric nature of the normal distribution,  $\sigma_\varepsilon^2$  can be correctly estimated from restricted data,  $D_j$  under our assumption.

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